DU Mathsoc Problem-Solving Set 16

March 24th, Hilary 2021

Problem 1: *A multiple choice examination consists of* 20 *questions. The scoring is* +5 *for each correct answer,* −2 *for each incorrect answer, and* 0 *for each unanswered question. John's score on the examination is* 48. *What is the maximum number of questions he could have answered correctly?*

(AJHSME 1987)

Problem 2: *The region consisting of all points in three-dimensional space within* 3 *units of line segment* $\overline{AB}$ *has volume* 216π. *What is the length* AB?

(AMC 10A 2017)

Problem 3: *In the multiplication problem below* A, B, C, D *are different digits. What is* $A + B$?

$$
\begin{array}{cccc}
 & A & B & A \\
\times & & C & D \\
\hline
C & D & C & D
\end{array}
$$

(AMC 8 2006)

Problem 4: *Suppose Ireland and Wales are equally strong at rugby. Which is more likely, that Ireland wins* 3 *games out of* 4, *or that Wales wins* 5 *games out of* 8? *(Ignore the possibility of draws.)*

(AHSME 1996)

Problem 5: *Find the number of ordered pairs of positive integers* (a, b) *such that* $a + b = 1000$ *and neither* a *nor* b *has a zero digit.*

(AIME 2006)

Problem 6: *Given an integer* $n \geq 2$, *find all positive solutions of the equation*

$$nx^{n+1} - (n+1)x^n + 1 = 0.$$

(Intervarsities Selection Test 2017)

Problem 7: *What are the last three digits of* 2009^{2009}?

(Intervarsities Selection Test 2009)

Problem 8: *Given a positive integer* n, *is the number*

$$a_n = \lfloor (2 + \sqrt{3})^n \rfloor$$

even or odd?

(Intervarsities Selection Test 2017)

Problem 9: *For which real numbers* x *does the sequence*

$$x, \ \sin x, \ \sin(\sin x), \ \sin(\sin(\sin x)), \ldots$$

converge?

(Intervarsities 2007)

Problem 10: *Let* A *be the* $n \times n$ *matrix, whose* $(i, j)^{th}$ *entry is* $i + j$ *for all* $i, j = 1, 2, \ldots, n$. *What is the rank of* A?

(IMC 2005)

Problem 11: *Let* $d_1, d_2, \ldots, d_{12}$ *be real numbers in the open interval* $(1, 12)$. *Show that there exist distinct indices* i, j, k *such that* d_i, d_j, d_k *are the side lengths of an acute triangle.*

(Putnam 2012 A1)

Problem 12: *What is the largest possible radius of a circle contained in a 4-dimensional hypercube of side length 1?*

(Putnam 2008 B3)