DU Mathsoc Problem-Solving Set 10

January 27th, Hilary Term 2021

Problem 1: *Corners a1 and h8 have been removed from a chessboard. Can you cover the rest with* 1×2 *dominoes?*

Problem 2: *A rook is on a1. Two players take turns moving the rook, and can only move it up or to the right. The player to reach h8 with the rook wins. Which player has the winning strategy (player who goes 1st or 2nd?).*

Problem 3: *What's the number of ways to place 8 rooks on a chessboard such that no rook is under attack?*

Problem 4: *In each unit square of an* 8×8 *chessboard we write one of the numbers* -1, 0 *or* 1. *Is it possible that all sums on rows, columns and the two diagonals are distinct?*

Problem 5: *Is it possible to move a knight from a1 to h8 by visiting each square on the chessboard exactly once?*

Problem 6: *Find the maximum number of specific chess pieces you can place on a chessboard such that none of them are under attack. Solve it for:*
(a) Rooks
(b) Queens
(c) Bishops
(d) Kings
(e) Knights

Problem 7: *Find a minimum number of specific chess pieces needed to place on a chessboard in a way that all free squares of the board are under attack. Solve it for:*
(a) Rooks
(b) Queens
(c) Bishops
(d) Kings
(e) Knights

Problem 8: *The crooked rook is a fictional chess piece that can only move one square up or one square to the right. Starting from the bottom left square of a standard chessboard (a1), how many ways are there to move the crooked rook to the top right square (h8), while avoiding the four squares in the center of the chessboard (d4, d5, e4, and e5)?*

(Misha Lavrov)

Problem 9: *Seven unit cells of an* 8×8 *chessboard are infected. In one time unit, the cells with at least two infected neighbours (having a common side) become infected. Can the infection spread to the whole square?*

Problem 10: *In how many ways is it possible to fill the squares of a chessboard with* -1 *and* 1 *such that the sum of elements in each* 2×2 *subarray is 0?*

(Columbia Math Olympiad)

Problem 11: *Every cell of a* 200×200 *table is coloured black or white. It is known that the difference between the number of black and white cells on the table is* 404. *Prove that some* 2×2 *square on the table contains an odd number of black unit squares.*

(Russia Math Olympiad 2000)

Chessboard for reference: