

DU Mathsoc Problem Solving

5th of October, Michaelmas 2020

P1. Find the probability that the ace of spades lies next to the jack of diamonds in an ordinary deck of 52 playing cards.

(Putnam 2014, B2)

(MAΘ 1991)

P2. Solve the system of equations

$$\begin{aligned} 2x_1 + x_2 + x_3 + x_4 + x_5 &= 6 \\ x_1 + 2x_2 + x_3 + x_4 + x_5 &= 12 \\ x_1 + x_2 + 2x_3 + x_4 + x_5 &= 24 \\ x_1 + x_2 + x_3 + 2x_4 + x_5 &= 48 \\ x_1 + x_2 + x_3 + x_4 + 2x_5 &= 96 \end{aligned}$$

(AIME 1986)

P3. What is the largest number which must evenly divide all integers of the form $n^5 - n$?

(AHSME 1957)

P4. Suppose a, b, c are complex numbers such that $a + b + c = 0$. Prove that

$$\begin{aligned} 2(a - b)^2(b - c)^2(c - a)^2 \\ = (a^2 + b^2 + c^2)^3 - 54a^2b^2c^2. \end{aligned}$$

(Finbarr Holland)

P5. Find all positive integers n for which $n^8 + n + 1$ is a prime number.

(Bernd Kreussler)

P6. Given a positive integer n , what is the largest k such that the numbers $1, 2, \dots, n$ can be put into k boxes so that the sum of the numbers in each box is the same? [When $n = 8$, the example $\{1, 2, 3, 6\}$, $\{4, 8\}$, $\{5, 7\}$ shows that the largest k is *at least* 3.]

(Putnam 2010, A1)

P7. Suppose f is a function on the interval $[1, 3]$ such that $-1 \leq f(x) \leq 1$ for all x and $\int_1^3 f(x)dx = 0$. How large can $\int_1^3 \frac{f(x)}{x} dx$ be?

P8. Prove that

$$\left\lfloor \frac{n + 2^0}{2^1} \right\rfloor + \left\lfloor \frac{n + 2^1}{2^2} \right\rfloor + \left\lfloor \frac{n + 2^2}{2^3} \right\rfloor + \dots + \left\lfloor \frac{n + 2^{n-1}}{2^n} \right\rfloor = n$$

for any positive integer n .

(The Art and Craft of Problem Solving)

P9. Let T be the set of all triples (a, b, c) of positive integers for which there exist triangles with side lengths a, b, c . Express

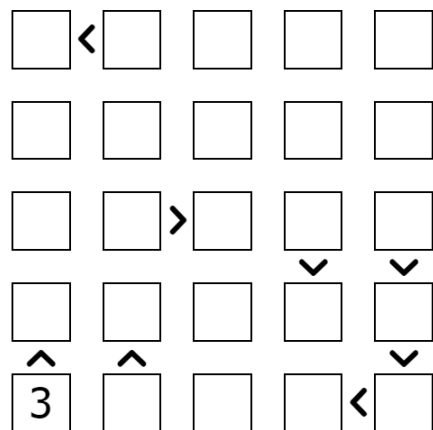
$$\sum_{(a,b,c) \in T} \frac{2^a}{3^b 5^c}$$

as a rational number in lowest terms.

(Putnam 2015, B4)

Futoshiki.

Fill the boxes with the numbers 1 to 5 so that each row and column contains exactly one instance of each number, and so that each inequality sign is respected.



Join the Mathsoc Discord server at 6pm on Thursday 8/10 for a discussion of this week's problems! Submissions, solutions and questions welcome: Darragh Glynn, quizmaster@mathsoc.com