Problem Solvers Week 8

1. If there are 2^n teams in a football tournament, and it's done by knockout, how many matches are there in the tournament?
2. Prove in every round robin tournament (everbody plays everybody) there is always a player p who for any other player q, has either beaten q or beaten someone who has beaten q.
3. Can you divide the face of a watch with 2 straight lines so that the sums of the numbers in each part are equal?
4. If you're trapped in a circular cage with somebody who can run at precisely the same speed as you, can you always catch them?
5. For what positive x is the x-th root of x the greatest?
6. Alice, Bob and Eve compete in a 100m race and run at constant speeds. Alice beats Bob by 10 metres and Bob beats Eve by 10 metres. How many metres does Alice beat Eve by?
7. Prove
$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n} = n2^{n-1}$$
8. In a lab, 2n students are present for a class, and the teacher needs to pair them up for performing a certain experiment. How many ways are there to do that?
9. Show that $\frac{(2n)!}{2^n n!}$ is an integer.
10. Show that $\frac{(kn)!}{k^n n!}$ is an integer.
11. Two men are located at opposite ends of a mountain range, at the same elevation. If the mountain range never drops below this starting elevation, is it possible for the two men to walk along the mountain range and reach each other's starting place, while always staying at the same elevation?
12. What is the maximum number of spots you can paint onto a football so that all the spots are the same distance from each other?
13. Prove that
$$\lim_{n\to\infty} n^2 \int_0^{1/n} x^{x+1} dx = 1/2$$