

# DU Mathsoc Problem Solving: Set 3

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~Advanced Paper~

For any assistance or inquiries about the problem sets, please email me at: [quizmaster.maths@csc.tcd.ie](mailto:quizmaster.maths@csc.tcd.ie).

**Problem 1.** Prove or disprove that there exists a positive real number  $u$  such that  $[u^n] - n$  is an even integer for all positive integers  $n$ . (Here,  $[x]$  is the greatest integer  $\leq x$ .)

**Problem 2.** Show that the next integer above  $(\sqrt{3} + 1)^{2n}$  is divisible by  $2^{n+1}$ .

**Problem 3.** Passengers  $P_1, P_2, \dots, P_n$  enter a plane with  $n$  seats. Each passenger has a different assigned seat. The first passenger sits in the wrong seat. Thereafter, each passenger either sits in their seat if unoccupied or otherwise sits in a random unoccupied seat. What is the probability that the last passenger sits in their own seat?

**Problem 4.** Can a  $2018 \times 2018$  grid be tiled with rotations and reflections of the  $L$ -tetromino?

**Problem 5.** For  $n \in \mathbb{N}$ , let  $x_n = \frac{1000^n}{n!}$ . Find the largest term of the sequence.