

# DU Mathsoc Problem Solving: Set 6

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~Beginners Paper~

For any assistance or inquiries about the problem sets, please email me at: [quizmaster.maths@csc.tcd.ie](mailto:quizmaster.maths@csc.tcd.ie).

**Problem 1.** Let  $n$  be a fixed positive integer. How many ways are there to write  $n$  as a sum of positive integers,  $n = a_1 + a_2 + \dots + a_k$ , with  $k$  an arbitrary positive integer and  $a_1 \leq a_2 \leq \dots \leq a_k \leq a_1 + 1$ ?

**Problem 2.** Given a positive integer  $n$ , what is the largest  $k$  such that the numbers  $1, 2, \dots, n$  can be put into  $k$  boxes such that the sum of the numbers in each box is the same?

**Problem 3.** Callie and Marie play a game in which they take turns filling entries of an initially empty  $2008 \times 2008$  array. Callie plays first. At each turn, a player chooses a real number and places it in a vacant entry. The game ends when all the entries are filled. Callie wins if the determinant of the resulting matrix is nonzero; Marie wins if it is zero. Which player has a winning strategy?

**Problem 4.** Let  $x$  be a natural number with the property that there exists a rearrangement of its digits such that the sum of  $x$  with the rearranged number is  $10^{2026}$ . Find the last digit of  $x^{2025}$ .

**Problem 5.** A  $7 \times 7$  square is covered by sixteen  $3 \times 1$  tiles and one  $1 \times 1$  tile. What are the possible positions of the  $1 \times 1$  tile?