DU Mathsoc Problem Solving: Set 8

Date: 11/11/2026

∼Advanced Paper∼

For any assistance or inquiries about the problem sets, please email me at: quizmaster.maths@csc.tcd.ie.

Problem 1. Prove that any whole number can be expressed in infinitely many ways as $\pm 1^2 \pm 3^2 \pm \ldots \pm (2k+1)^2$ for some choice of the plus and minus sign sequence and k a natural number.

Problem 2. Consider a point M on a sphere that's inscribed in a cube. Show that the sum of the squared distances from M to the 6 faces of the cube is constant.

Problem 3. Let n balls be thrown uniformly and independently into n bins. Find the probability that bin 1 is empty and the expected number of empty bins.

Problem 4. Show that no three points in the $\mathbb{Z}^2$ plane can be the vertices of an equilateral triangle. What about in the $\mathbb{Z}^3$ space?

Problem 5. Show that there exists an equiangular polygon with side lengths $1^2, 2^2, \ldots, 1990^2$ (not necessarily in this order).